How Hedge Funds Fit

The quest for institutional quality.

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One explanation for the rapid rise of interest in hedge funds is that institutional investors believe there is something about the expected performance of hedge funds that they cannot find elsewhere. Yet this is surprising, as most hedge funds have an opportunity set that is no more extensive than that employed by traditional managers. What is so different about hedge funds?

Hedge funds differ from traditional investments in many respects including benchmarks, investment processes, fees, and regulatory environment. Traditional active managers are usually benchmarked to a well-understood market index. Hedge funds, however, are often claimed as *absolute return strategies*, implying that their performance should be compared with a reference return of zero; more sophisticated investors understand that the benchmark of these funds is *intended* to be cash.

However we characterize their performance, hedge funds claim extraordinary levels of investment skill, building strong performance expectations. These expectations are used to justify high base and incentive fees.

Aside from benchmarks and fee structures, a further difference is that hedge funds are typically allowed to hold securities long or short, while traditional strategies are required to be long-only. This freedom from the long-only constraint is a material advantage for hedge funds over traditional investments. For equal skill and risk budgets, it has been shown that the expected performance of a long-only equity manager may be roughly doubled—for better or worse—by allowing the same manager to be unconstrained long and short (see Clarke, de Silva, and Thorley [2002] and Grinold and Kahn [2000]).

Ultimately, the rush to hedge funds could be interpreted as a trend of institutional investors seeking more alpha and perhaps even less beta. This is a phenomenon that cannot be encouraged without extreme caution because the expectation of positive alpha—for any strategy—is conditional on manager skill. Furthermore, it requires additional investor skill to evaluate the quality of hedge funds and to know how a hedge fund fits into the portfolio. Indeed, hedge funds do not fit unless and until one can understand how their strategies affect the performance of the entire portfolio, in both its beta and alpha characteristics. This is possible only with what we will define as *institutional quality* hedge funds.

We begin with a discussion of how to separate alpha (specific exposures) from beta (systematic exposures) in hedge fund returns to evaluate a hedge fund's normal portfolio. Systematic exposures can be found by analyzing the historical performance of hedge fund index returns, but for most hedge funds this leads to several complications. To resolve these issues, minimum conditions for institutional quality are identified.

To address the central question—how hedge funds fit into the portfolio—optimal rules for hedge fund allocation are demonstrated, given an investor's constant risk aversion or constant risk budget objectives. Two approaches for incorporating hedge funds are demonstrated: 1) hedge funds as an alpha overlay at the total portfolio level; and 2) hedge funds as portable alpha within a traditional asset class.

SEPARATING ALPHA AND BETA

We should think of hedge fund returns as composed of systematic and idiosyncratic components, even though both components may be more complex for hedge funds than for traditional products. Understanding the components of alpha and beta in hedge fund returns is important for several reasons. As we have noted, we must separate the components to rationalize fee structures. Isolating the components is crucial to understanding how an allocation to hedge funds impacts overall portfolio performance and thus how hedge funds fit into the portfolio.

Various academicians and practitioners have investigated hedge fund correlations, which is a first step in separating alpha and beta. The statistically significant correlations observed between hedge fund styles indicate the presence of common factors driving hedge fund returns, including traditional asset class factors. See, for example, Fung and Hsieh [2004], Goetzmann and Brown [2003], and Schneeweis and Spurgin [1998]. Ultimately, these observations may be constructive in defining the *normal portfolio* of any hedge fund.

Defining the Normal Portfolio

The single-factor market model is commonly represented as follows, here subscripted in a manner appropriate for its application to a hedge fund:

$$\tilde{r}_{hf} = \tilde{\alpha}_{hf} + \beta_{hf}\tilde{r}_b \tag{1}$$

where $\tilde{r}_{hf}$ is the hedge fund return in excess of the risk-free rate, $\tilde{\alpha}_{hf}$ is the residual return of the hedge fund, β_{hf} is the systematic risk factor of the hedge fund, and $\tilde{r}_b$ is the return of a market index (benchmark) in excess of the risk-free rate. The notation $\tilde{r}$ signifies that r is a random variable.

The value of beta may be estimated as:

$$\beta_{hf} = \rho\frac{\sigma_{hf}}{\sigma_b} \tag{2}$$

where ρ is the hedge fund-market index correlation, σ_{hf} is the volatility of the hedge fund, and σ_b is the volatility of the market index (benchmark).

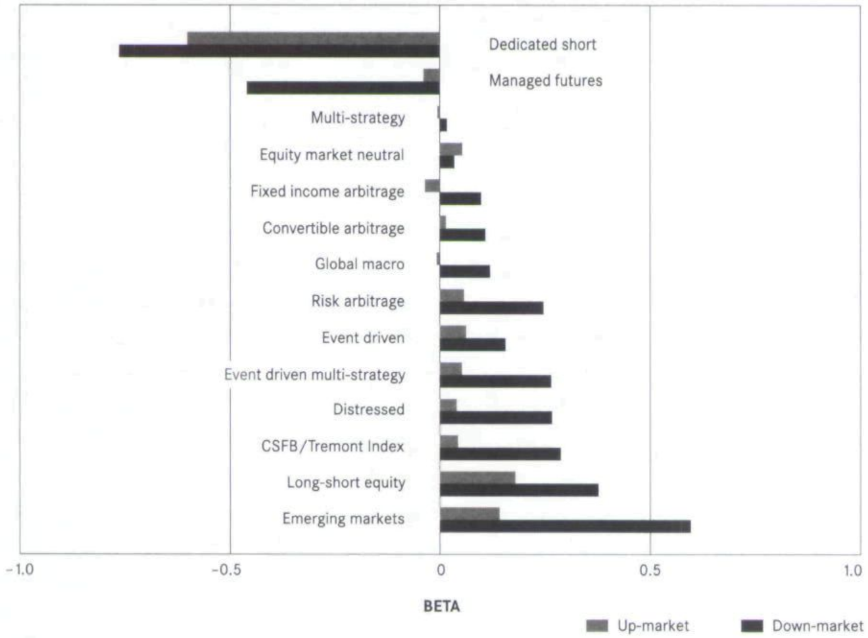
This familiar framework reminds us that this sort of analysis, evaluating the normal portfolio, ought to apply to hedge fund strategies just as it applies to any other manager. Systematic and active exposures are separated by first estimating beta, as in Equation (2), and then subtracting the systematic return to estimate alpha, as in Equation (1).

The factor model is highly simplified here, as hedge fund returns often show evidence of multifactor market exposures so that $\tilde{r}_b$ and β_{hf} may represent vector rather than scalar quantities (see Asness, Krail, and Liew [2001]). Even so, it can be used as a first step to evaluate performance characteristics of hedge funds.

Complications for Hedge Funds

An impediment to using Equations (1) and (2) is that the correlations and volatilities of hedge funds are not fixed but instead may differ depending on market direction and market states. These complications imply that the beta

Exhibit 1
Asymmetric Hedge Fund Betas



Realized beta estimates for up and down markets relative to Russell 3000 Index based on monthly returns of CSFB/Tremont hedge fund style indexes, 1994-2004.

Source: CSFB/Tremont, as of 12/31/04.

coefficient associated with hedge funds is not a single constant but rather much more complex. As hedge funds involve derivative investments and leverage, it is not surprising to observe this complexity in the beta exposure.

For example, there appears to be an asymmetric dependence between the realized monthly returns of the CSFB/Tremont Hedge Fund Index and the returns of the Russell 3000 Index. We see in the recent ten years of returns for the CSFB/Tremont index that the realized beta for down markets was 0.30 while the beta for up markets was only 0.04. A down market is here defined as a monthly period when the Russell 3000 has a negative return, and an up market is defined conversely as a monthly period when the Russell 3000 has a positive return. In other words, the index shows *asymmetric beta* exposures.[1]

The CSFB/Tremont index represents a composite of hundreds of hedge funds, representing realized beta exposures for a highly diversified portfolio of hedge funds. Hedge funds are usually classified into styles sharing similar investment characteristics that may differ in beta exposures from the composite.

Exhibit 1 shows realized beta estimates for 13 hedge fund styles plus the CSFB/Tremont index in order to observe any style-related asymmetric beta exposures. The realized betas are calculated separately for up and down monthly returns relative to the Russell 3000 Index.

We see asymmetric negative betas for dedicated short and managed futures styles. The betas associated with multistrategy and equity market-neutral styles are insignificant, as expected. The remaining hedge fund styles all have asymmetric betas, with the pattern of a higher beta in the down market scenario than in the up market scenario.

If this pattern of asymmetric betas is representative of a forward-looking view, Equation (1) needs to be rewritten as follows for many hedge fund styles:

$$\tilde{r}_{hf} = \tilde{\alpha}_{hf} + \beta_{+}\tilde{r}_{b+} + \beta_{-}\tilde{r}_{b-} \qquad (3)$$

where β_{+} is the up market beta conditional on $r_b > 0$, β_{-} is the down market beta conditional on $r_b \leq 0$, and r_{b+} and r_{b-} are the conditional up and down market distributions of the index. This is similar to an approach Lo [2001] suggests to deal with nonlinearities in hedge fund index returns.

Equation (3) expresses the asymmetric beta estimates for hedge fund styles observed in Exhibit 1. While traditional active managers have a well-defined normal portfolio, this model suggests that many hedge funds do not have a simple normal portfolio with respect to ordinary market factors. Traditional active managers engaging in market timing activity cause (symmetric) variability of the realized beta surrounding an expected beta, or normal portfolio. The statistical fit of Equation (1) may be diminished depending on the extent of market timing activity, but the single-factor model remains a valid representation of systematic exposures as separate from active exposures.

This is distinguished from the systematic exposures for hedge funds that have been shown to involve complex (asymmetric) option-like characteristics. As a result, for many hedge funds there is not a simple normal portfolio expressible in terms of ordinary and non-option market factors as there is for traditional active managers.

And the systematic exposures may be even more varied and complex than identified in Exhibit 1.

Generalizing Equation (3), hedge fund returns may be represented as:

$$\tilde{r}_{hf} = \tilde{\alpha}_{hf} + \tilde{\beta}_{hf}\tilde{r}_b \tag{4}$$

where the beta is a state-dependent random variable conditional on up and down states of the market as in (3), or even more complex. Equation (4) is consistent with the observation that hedge fund returns are not normally distributed like most traditional manager returns.[2]

This discussion should illustrate the difficulty of trying to estimate normal portfolios for hedge funds, and this is a great barrier to their use in institutional portfolios. Not understanding the hedge fund's normal portfolio means we cannot separate alpha from beta, which causes further difficulty in developing confidence that the manager can produce a positive expected pure alpha as opposed to complex beta exposures passed off as alpha.

If we are unable to resolve these issues, it is impossible to understand how hedge funds impact overall portfolio performance. For these reasons, it is necessary to define the subset of strategies that do not suffer from these deficiencies, referred to as institutional quality hedge funds.

INSTITUTIONAL QUALITY HEDGE FUNDS

Hedge funds have certainly influenced the investing behavior of institutional investors, but it is interesting to ponder whether the needs of institutional investors will change the practices of hedge fund managers. In other words, will the investment characteristics of the best hedge funds change to meet an institutional quality standard?

There are minimum standards to which hedge funds should be held before they are given serious consideration in the professionally managed institutional portfolio. For a hedge fund to be institutional quality, the hedge fund *and the investor* must meet requirements as follows:

1. Clear identification by the investor of the hedge fund's normal portfolio, providing an unambiguous separation of alpha and beta from total returns.
2. Skillful judgment by the investor that the hedge fund manager has the ability to produce a positive expected pure alpha.

These criteria apply both to the candidate hedge funds and to the selection process of the investor. The first requirement ensures that a normal portfolio for the hedge fund is carefully estimated and understood. The normal portfolio separates the return associated with systematic exposures from residual exposures. By identifying the systematic exposures, the investor can understand how an allocation to the hedge fund impacts overall portfolio risk via the risk of the hedge fund itself and covariance with other allocations. Further, the fee structure can be rationalized, as noted earlier.

The first condition also prohibits the asymmetric beta exposures that have been observed for hedge fund style indexes.[3] It is acceptable to have a beta exposure, but the presence of asymmetric or other complex beta exposures is undesirable and complicates our ability to incorporate such an investment in the overall portfolio. Satisfying the first requirement resolves the issues we have identified concerning the definitive separation of alpha and beta.

The second requirement concerns skill. This skill requirement is not exclusive to hedge funds but applies to incorporating *any* active strategy in a portfolio. This point is worth repeating here because investors tend to avoid the question of skill in hedge fund investing. It is the most critical question, because the ultimate goal of investing in hedge funds is to find a positive expected pure alpha, which is possible only with above-average skill. This criterion is especially important now because of the rapidly growing flows of institutional assets into the hedge fund market and the increasing competition for talent and ideas.

These two requirements are neither a beginning nor an end to a debate on what constitutes institutional quality. A dominant theme of the hedge fund literature, for example, has been the issue of transparency of hedge fund strategies, referring to the problem of understanding the inner workings of a fund. Clearly, hedge fund managers need to protect any proprietary insights of their strategies, and they also need to protect information on current positions against others who would take advantage. Institutional investors, on the other hand, require and are accustomed to fairly complete disclosure. Therefore, institutional investors should favor hedge funds that explicitly describe the sources of systematic and active exposures.

Another theme in the literature has been the question of benchmarks, or a relative-return framework, although this is not often linked to the issue of institutional quality. One exception is Anson [2001], who speaks of institutional needs for a relative-return framework in addition to transparency and a well-defined investment process. Asness [2004] also notes that hedge funds need to facilitate institutionalization with regard to benchmarking and articulation of strategies in terms of alpha and beta.

Waring and Siegel [2005] state that all forms of active management, including hedge funds, consist of making bets relative to some sort of benchmark, regardless of any claims to the contrary including the claim implicit in the term, "absolute returns." Casey, Quirk, and Acito [2004] observe that institutional investors require risk control, a concept that ought to address characteristics of a hedge fund's normal portfolio and idiosyncratic risks. Inevitably, institutional investors must evaluate hedge fund performance in a relative-return framework and must understand hedge fund normal portfolios in order to assemble portfolios of managers.

To sum this up, the separation of alpha and beta and the identification of skill are concise criteria that are clearly essential to make a professional decision about incorporating hedge funds into portfolios. With these criteria, institutional investors are better able to sort out hedge fund managers of institutional quality from those that are not. After identifying institutional quality hedge fund candidates, investors can determine the impact of allocations to these funds on the overall portfolio. Then the question turns to how much allocation and how it fits into the portfolio.

HOW HEDGE FUNDS FIT

If we can select hedge funds that satisfy the conditions for institutional quality, we can move on to the principles for allocation and consider approaches to fitting institutional quality hedge funds into institutional portfolios.

Allocation to Institutional Quality Hedge Funds

Having determined that the candidate hedge fund is institutional quality, modern portfolio theory and the principles of active management can be applied to establish an optimal allocation. The requirements for institutional quality include a clear understanding of the normal portfolio and the residual component of returns.

For simplicity, the asset class or planwide policy benchmark is assumed to be $\tilde{r}_b$, the same market index (scalar or vector) that is used to characterize the normal portfolio of a hedge fund in Equation (1). Then, the total active return of the institutional quality hedge fund relative to the policy benchmark may be expressed as:

$$\tilde{r}_{hf} - \tilde{r}_b = \tilde{\alpha}_{hf} + (\beta_{hf} - 1)\tilde{r}_b \tag{5}$$

From (5), the active risk of the hedge fund σ_{hf} is determined by the relationship:

$$\sigma_{hf}^2 = \omega_{hf}^2 + (\beta_{hf} - 1)^2 \sigma_b^2 \tag{6}$$

where ω_{hf} is the non-systematic risk component, and σ_b is the volatility of the benchmark.

The first term on the right-hand side of (6) is the pure active variance, and the second term is the misfit variance caused by the hedge fund's misfit beta exposure. *Misfit beta exposure* occurs because the hedge fund's systematic exposure is different from policy; when the beta exposure of the hedge fund is the same as the benchmark, the misfit variance term disappears.

Naturally, the measurement of hedge fund active risk depends on the benchmark that it is measured against. S*tated* benchmarks do not matter; what matters is the hedge fund's active risk relative to its normal portfolio and how that normal portfolio compares to the investor's policy. If the systematic exposure of the hedge fund is known and stable (i.e., institutional quality), it is possible to reduce or eliminate this effect.

Assume the investor believes the hedge fund has a positive expected alpha of α_{hf} and that active risk contributed by the hedge fund is σ_{hf}. The consistency of performance is measured by the expected information ratio, $IR_{hf} = \alpha_{hf}/\sigma_{hf}$, read as *expected hedge fund alpha divided by hedge fund active risk.*

Assuming that an investor's starting portfolio optimizes investor utility with respect to asset allocation and manager structure, the optimal portion of the portfolio that should be put into hedge funds, while maintaining the same risk aversion, may be approximated as:

$$h = \frac{IR_{hf} / \sigma_{hf}}{IR_0 / \sigma_0} \tag{7}$$

where $IR_0 = \alpha_0/\sigma_0$ is the expected information ratio of the initial portfolio before adding the hedge fund, read as *expected initial alpha divided by initial active risk.*

In this instance, while the risk aversion is held constant, the investor's risk budget increases in the presence of an attractive hedge fund opportunity. Some investors may wish to maintain the same active risk budget embedded in the current manager profile. For a constant risk budget, the optimal allocation to the hedge fund would be revised to:

$$h = \frac{IR_{hf} / \sigma_{hf}}{\sqrt{IR_{hf}^2 + IR_0^2} / \sigma_0} \qquad (8)$$

Simply put, these relationships tell us that the optimal allocation to a hedge fund is proportional to the expected information ratio of the hedge fund and inversely proportional to the active risk of the hedge fund. A derivation of Equations (7) and (8) is in the appendix.

The active risk of the hedge fund that is an ingredient of Equations (7) and (8) includes the volatility associated with both the non-systematic and misfit beta components of the hedge fund return stream. As long as the hedge fund is institutional quality, by definition, the normal portfolio should be well understood, and misfit beta exposure could be adjusted by equitization so as not to affect strategic asset allocation policy.

If not, the two formulas both demonstrate how the optimal allocation to the hedge fund would be reduced by an uncorrected misfit beta exposure. Specifically, an increase in the absolute value of misfit beta ($\beta_{hf} - 1$) increases hedge fund active risk and reduces the optimal hedge fund allocation.

As an example, assume the investor has a current active risk budget of $\sigma_0 = 1.0\%$ portfoliowide, with an expected alpha of $\alpha_0 = 0.5\%$, implying an expected information ratio of $IR_0 = 0.50$, values that many institutional investors would view as typical, presuming skill. Further assume that in the considered opinion of the investor, the expected pure alpha of the hedge fund is a very skillful 7.5% and the active risk 10.0%, so that the expected information ratio is $IR_{hf} = 0.75$.

By calculation using Equations (7) and (8), the optimal allocation to a hedge fund is approximately 15.0% for constant risk aversion and 8.3% for a constant risk budget. These calculations assume the normal portfolio beta exposure for the investment has been adjusted so as not to affect the investor's current policy benchmark, in this instance the Russell 3000.

If instead the hedge fund had a systematic exposure typical of, say, the CSFB/Tremont index, and this misfit were not corrected, the optimal allocation to the hedge fund would be reduced by 2.8 percentage points to 12.2% for constant risk aversion and by 1.1 percentage points to 7.2% for a constant risk budget. In this instance, the hedge fund allocation produces a misfit risk, based on a single-factor model of CSFB/Tremont hedge fund returns with an unadjusted beta of +0.3 relative to the equity index that has 16% annual volatility. The resulting misfit risk of approximately 4.8 percentage points increases hedge fund active risk from 10.0% to 11.1%. The optimal allocations are thereby reduced, reflecting the lower expected information ratio and the higher active risk of the hedge fund.

These formulas and examples demonstrate important principles that support our intuition in allocating to institutional quality hedge funds. First, the allocation depends directly on the expected information ratio of the hedge fund relative to the information ratio of the starting portfolio. Second, the allocation is inversely related to hedge fund risk. Further, "surprise" beta exposures that go uncorrected increase hedge fund risk and reduce the optimal allocation.

Hedge Funds in the SAA Policy?

The most commonly used approach to allocation is to consider hedge funds as part of the strategic asset allocation process. Without a clear idea of where to put them, investors usually lump hedge funds in the alternative asset class. Investors often establish an optimistic target return for an alternative asset class that implicitly incorporates a significant positive expected alpha. Thus, the hedge fund beta and alpha are mixed, although we understand that our risk preferences toward alpha and beta—and our costs for alpha and beta—are very, very different.

The strategic asset allocation process normally deals only with asset class beta exposures defined by fully diversified market segments. But hedge funds are *not* a true asset class; in this approach, we are just trying to force them into this framework.

In addition to a problem with handling alpha in this framework, there may also be a problem with handling beta. The hedge fund beta exposures signal correlation with market exposures across other asset classes in a complex way that varies by style and by manager. In reality, one may be able to estimate the beta exposures of a hedge fund allocation only after identifying the specific managers and analyzing their unique exposures.

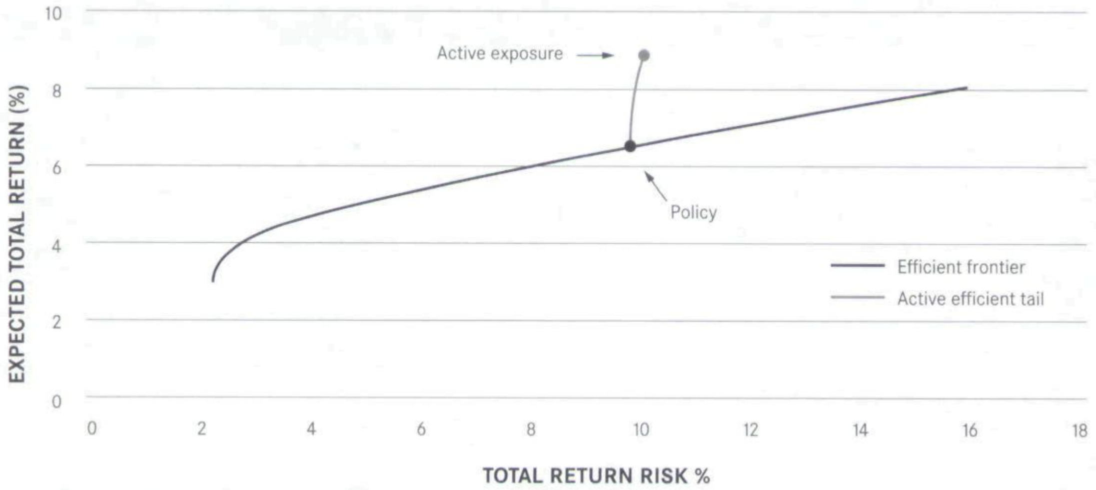
Two approaches are recommended for including hedge funds in the portfolio:

1. As an *active overlay* at the total portfolio level.
2. As *portable alpha* within a traditional asset class.

Approach 1: Active Overlay at the Total Portfolio Level

The first approach is to consider the expected alpha and active risk generated by hedge funds on a portfo-

EXHIBIT 2
Hedge Funds as Active Overlay at Total Plan Level



liowide basis. This is a variation on what may be called an active overlay approach.

In Exhibit 2, the lower curve represents the familiar efficient frontier, the expected return and risk associated with systematic exposures across all asset classes. The active efficient tail is rooted at the overall policy and contributes additional total return and risk associated with the active exposures of the hedge fund.

This framework properly distinguishes alpha from beta, assuming we know how much of each is contributed by the hedge fund allocation. The combination of the chosen policy and the chosen active exposure gives the expected total return and total risk.

The caution in this instance is to ensure that any normal portfolio beta exposure associated with a hedge fund allocation is corrected so as to not to change the SAA policy decision. The beta exposures associated with the institutional quality hedge fund, once known, can be adjusted using equitization or other means to align with the overall beta target of the portfolio.

When we implement this adjustment to maintain the original SAA policy, it is referred to as beta transfer or portable beta. Even without a full portable beta approach, it is valuable to think about hedge funds as potential contributors to total portfolio alpha. In reality, alpha is a return stream that is technically independent of any asset class, and the investor can think of all alpha sources as if they were at the total portfolio level.

Exhibit 3 shows how much alpha and how much beta results from a 10% allocation to a hedge fund. The expected alpha for the hedge fund is 7.5%, a value that presumes special skill, and the active risk is 10.00%. A 10.00% allocation gives 0.75% expected alpha and 1.00% active risk, as shown.

The alpha adds directly to expected total return, but the hedge fund active risk adds only incremental risk to total portfolio risk because uncorrelated risks add in the sum of squares (variances). In fact, the 1.00% active risk adds only 0.05% to total return risk in this example. While total risk is very low in this instance, one must be careful about thinking in total risk terms as many investors have a higher aversion to active risk than to policy risk.

Having demonstrated that an allocation to an institutional quality hedge fund improves portfolio performance (conditional on skill), one needs to assess an appropriate allocation level, determined above by Equations (7) or (8). As always, the investor must have confidence that these estimates are justified by the skill of the manager.

The positive alpha tail sits atop the efficient frontier in Exhibit 2 and is additive to total returns only because we are assuming we have special skill in evaluating these managers. But if we are wrong about our assessment of special skill, the true but unplotted tail will be between our legs (literally, sloping downward).

Approach 2: Portable Alpha Within an Asset Class

The second approach is to evaluate the hedge fund in one of the traditional asset classes. In this approach we

EXHIBIT 3
Impact of 10% Hedge Fund Overlay on Total Portfolio Return and Risk

		Expected return (%)	Expected risk (%)
	Hedge fund alpha	7.50	10.00
Impact	Active overlay (alpha)	0.75	1.00
Impact	Original portfolio (beta)	6.50	9.75
Impact	Final portfolio	7.25	9.80

EXHIBIT 4
Manager Assumptions

	Expected alpha (%)	Expected risk (%)	Current holdings (%)
Large-cap index	0.00	0.00	45.0
Small-cap index	0.00	0.00	0.0
Large-cap enhanced	1.00	2.00	20.0
Large growth active	2.00	7.00	10.0
Large value active	1.75	6.00	10.0
Small growth active	2.00	7.00	7.5
Small value active	2.25	7.00	7.5
Hedge fund (equitized)	**7.50**	**10.00**	**0.0**
Total	0.89	1.25	1.00

use a *manager structure optimization* methodology to determine the optimal allocation to hedge funds.[4]

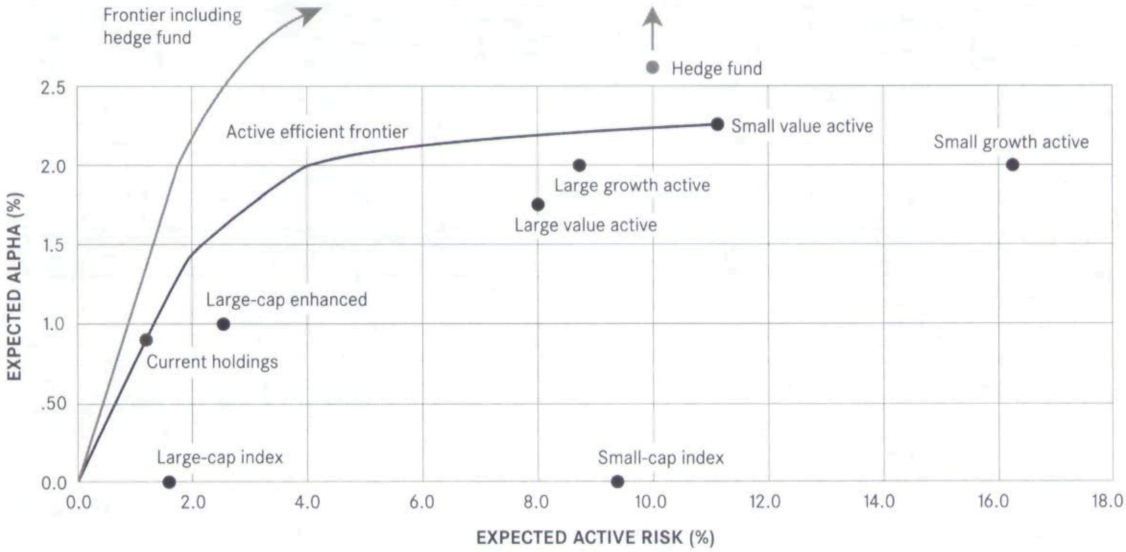
A distinction between the two approaches is that approach 1 uses the total portfolio SAA as a benchmark and approach 2 uses a single asset class benchmark. We can illustrate this difference with a case example.

Exhibit 4 shows a list of managers for a hypothetical domestic equity portfolio: large-cap, small-cap, value and growth, indexed, enhanced and active. Initially there is no allocation to hedge funds, but we are considering the same hedge fund example that is benchmarked to cash that has an expected alpha of 7.5% and a risk level of 10.0%.

The active risk level seems very high to add to the equity portfolio, but we are able to control risk by adjusting the size of the allocation and by performing a beta transfer on the hedge fund to match the equity asset class benchmark. This *equitization* of the hedge fund can be optimized by trading off tracking error and transaction cost for the various beta transfer alternatives (e.g., futures, swaps).

Exhibit 5 shows the individual managers plotted in active space, with expected alpha plotted against expected active risk (relative to the portfolio benchmark), and an active efficient frontier. Each manager is treated like a security in a mean-variance framework. The current holdings portfolio has an expected alpha of 0.89% with an active risk budget of 1.25%, and lies on the active efficient frontier before consideration of the hedge fund. The hedge

EXHIBIT 5
Hedge Funds as Portable Alpha Within an Asset Class



fund itself would plot up, off the chart.

The exhibit also shows how incorporating institutional quality hedge funds dramatically elevates the possibilities, represented by a new active efficient frontier that incorporates optimal allocations to the hedge fund. The optimal allocation to the hedge fund derived from the new frontier is shown in Exhibit 6 at various risk budgets.

Taking the current holdings risk budget of 1.25%, the optimal allocation to the hedge fund is 8.70%. At lower risk, the optimal allocation to the hedge fund is small, around 3.00%. At higher risk budgets—typical of endowments and foundations—there would be a higher allocation in the range of 10.00%–14.00%.

It is important to remember that this level of enhancement is possible only for hedge funds that have satisfied the conditions for institutional quality. It presumes that the professionals who oversee the equity asset class have the skills and the experience to evaluate the candidate hedge funds, although many hedge funds do not fit neatly into a single asset class.

This within-the-asset-class approach has the advantage of building on the investor's revealed preferences. In other words, the current holdings reveal the investor's preferences for an active risk budget and the desired trade-off of risk and expected return. An allocation to hedge funds can be made to fit within the current active risk budget or, alternatively, an allocation that is consistent with the investor's degree of risk aversion implied by current holdings.

CONCLUSION

The first challenge for fitting hedge funds into the portfolio is to distinguish hedge funds that are institutional quality from those that are not. For example, it has been emphasized that a hedge fund lacking transparency or just repackaging beta disguised as alpha is not appropriate for professional institutional management. Instead, institutional quality means there is clarity on the systematic exposures present in the strategy, and there is confidence that the hedge fund can produce a positive expected alpha.

The next challenge is to make an appropriate allocation. Since hedge funds are not a true asset class, following the most common approach used by investors—allocating assets as if these funds were a separate asset class—is clearly inconsistent.

While there is no single right answer as to how hedge funds fit in an institutional portfolio, there are two theoretically sound approaches for incorporating these investment strategies. Using an alpha overlay framework at the total portfolio level or considering the equitized hedge fund in a traditional asset class is consistent with modern portfolio theory and with the theory of active management.

Ultimately, what matters to the investor is the impact of hedge funds on overall portfolio return and risk and, for an institutional investor, this determination ought to be part of an institutional quality process.

EXHIBIT 6
Optimal Allocation to Hedge Funds Depends on Active Risk Budget

	Current holdings	Optimal mixes					
Active risk budget (%)	1.25	0.00	0.50	1.00	1.25	1.50	2.00
Expected alpha (%)	0.89	0.00	0.54	1.08	1.35	1.62	2.16
Information ratio	0.71	-	1.08	1.08	1.08	1.08	1.08
Manager allocations (%)							
Large-cap index	45.0	86.0	67.0	47.5	38.2	28.5	7.3
Small-cap index	0.0	14.0	10.0	6.0	4.0	2.0	0.0
Large-cap enhanced	20.0	0.0	11.5	23.2	28.9	34.7	47.3
Large growth active	10.0	0.0	2.0	4.0	4.9	5.9	8.1
Large value active	10.0	0.0	2.1	4.3	5.3	6.4	8.6
Small growth active	7.5	0.0	2.0	4.0	4.9	5.9	7.0
Small value active	7.5	0.0	2.0	4.1	5.1	6.1	7.5
Hedge fund (equitized)	0.0	0.0	3.4	7.0	8.7	10.4	14.2

APPENDIX

Derivations

This appendix summarizes the derivation of Equations (7) and (8), the optimal allocation to institutional quality hedge funds.

Investor's Objective Function

The mean-variance objective function for the investor follows the development in Grinold and Kahn [2000, pp. 105–108], and Waring et al. [2000, Appendix A]. The expected total return of a portfolio in excess of the risk-free rate is:

$$r = \underbrace{(1 - h_1 - h)r_b}_{benchmarkindex} + \underbrace{h_1(\alpha_1 + r_b)}_{existingactivemgrs} + \underbrace{\mu h(\alpha_{hf} + \beta_{hf} r_b)}_{hedge\,fund}$$
$$= r_b + h_1\alpha_1 + h(\alpha_{hf} + (\beta_{hf} - 1)r_b) \quad \text{(A-1)}$$

where h_1 is the allocation to the current structure of traditional active strategies, h is the allocation to the hedge fund, and $(1 - h_1 - h)$ is the allocation to passive index strategies. The traditional active and passive strategies are assumed to be on benchmark.

The hedge fund returns are assumed to have systematic and active components as in Equation (1), $r_{hf} = \alpha_{hf} + \beta_{hf} r_b$, where β_{hf} is the systematic risk factor of the hedge fund relative to the benchmark. The variance of total return is then:

$$\sigma^2 = \underbrace{\sigma_b^2}_{benchmark} + \underbrace{h_1^2\omega_1^2}_{existingactivemgrs} + \underbrace{h^2\sigma_{hf}^2 + 2h(\beta_{hf} - 1)\sigma_b^2}_{hedge\,fund} . \quad \text{(A-2)}$$

where σ_b^2 is the volatility of the benchmark, ω_1^2 is the volatility of the traditional active strategy, and $\sigma_{hf}^2 = \omega_{hf}^2 + (\beta_{hf} - 1)^2\sigma_b^2$ is the hedge fund's active volatility, incorporating both residual risk and misfit beta risk.

The objective function trades off expected return against risk, but here we allow for different risk penalties associated with different types of risk. We assume λ is the risk penalty associated with active risk, and λ_b is the risk penalty associated with market risk and the covariance term. Thus, the overall objective function is to maximize:

$$U = \underbrace{r_b - \lambda_b\sigma_b^2}_{SAA} + \underbrace{h_1\alpha_1 + h(\alpha_{hf} + (\beta_{hf} - 1)r_b) - \lambda(h_1^2\omega_1^2 + h^2\sigma_{hf}^2) - 2\lambda_b h(\beta_{hf} - 1)\sigma_b^2}_{MSO} \quad \text{(A-3)}$$

The terms relating to the strategic asset allocation (SAA) objective are separated from the terms for the manager structure optimization (MSO) objective.

Since the MSO problem optimizes the manager holdings separate from the SAA problem, for our purposes we can focus solely on the MSO components from the objective function. Further, by using an equilibrium condition for λ_b, namely, $r_b = 2\lambda_b\sigma_b^2$, the MSO objective function simplifies to:

$$U_a = h_1\alpha_1 + h\alpha_{hf} - \lambda(h_1^2\omega_1^2 + h^2\sigma_{hf}^2) \tag{A-4}$$

The expected misfit return and the covariance term are dropped, although the impact of misfit risk remains a component of the expected utility within the hedge fund variance σ_{hf}^2.

In words, the MSO objective (in the absence of exceptional forecasts) is to maximize expected pure alpha, subject to total active variance. Active variance includes both traditional active and hedge fund residual risk plus misfit risk associated with the uncorrected hedge fund beta.

Optimal Allocation: Constant Risk Aversion

We assume there is a starting portfolio that combines an allocation of h_1 to active strategies and $(1 - h_1)$ to passive index strategies. Then, the active variance budget is defined by $\sigma_o^2 = (1 - h_1)^2 0 + h_1^2\omega_1^2$, and the active return is $\alpha_o = (1 - h_1)0 + h_1\alpha_1$, where α_1 is the return of active strategies, and ω_1 is the associated risk. The information ratio of the starting portfolio is $IR_o \equiv \alpha_o/\sigma_o = \alpha_1/\omega_1$.

If the initial allocation is optimal without a hedge fund allocation, then the first-order conditions for mean-variance optimization imply $2\lambda_o = \alpha_1/\omega_1^2$. Now, adding a hedge fund, our objective is to maximize U_a using $\lambda = \lambda_0$ for the investor's (constant) risk aversion parameter. The first-order conditions imply:

$$h = \frac{\alpha_{hf}}{2\lambda_o\sigma_{hf}^2} = \frac{\alpha_{hf}/\sigma_{hf}^2}{\alpha_o/\omega_o^2}$$

Then Equation (7) follows:

$$h = \frac{IR_{hf}/\sigma_{hf}}{IR_o/\sigma_o}$$

Optimal Allocation: Constant Risk Budget

The starting risk budget variance σ_o^2 is equated with the active variance of a new portfolio that includes a hedge fund allocation: $\sigma_o^2 = h_1^2\omega_1^2 + h^2\sigma_{hf}^2$. Rearranging, a constant active risk budget requires $h_1 = (\sigma_o^2 - h^2\sigma_{hf}^2)^{1/2}/\omega_1$. By substitution, we have a goal to maximize $h_1\alpha_1 + h\alpha_{hf} = (\sigma_o^2 - h^2\sigma_{hf}^2)^{1/2}\alpha_1\omega_1 + h\alpha_{hf}$, implying the first-order conditions:

$$-h\sigma_{hf}^2(\sigma_o^2 - h^2\sigma_{hf}^2)^{-1/2}\alpha_1/\omega_1 + \alpha_{hf} = 0$$

Rearranging:

$$h^2 = \left(\frac{\sigma_o^2}{\sigma_{hf}^2}\right)\frac{(\alpha_{hf}/\sigma_{hf})^2}{(\alpha_1/\omega_1)^2 + (\alpha_{hf}/\sigma_{hf})^2}$$

Then Equation (8) follows:

$$h = \frac{IR_{hf}/\sigma_{hf}}{\sqrt{IR_{hf}^2 + IR_o^2}/\sigma_o}$$

This formula is an approximation because it assumes that the information ratio of the initial portfolio is constant for small changes in the risk level. For typical portfolios, there is a marginal increase in the expected information ratio of optimal portfolios at smaller risk budgets.

ENDNOTES

The author thanks his colleagues for their insights and encouragement, especially Barton Waring, who has provided numerous comments that have greatly benefited this article. Thanks also to Mark Anson for helpful comments and suggestions.

[1]Consolidated hedge fund performance looks a bit like a short option on the market. This short option strategy exposure pattern can be observed with many hedge fund types, but it may be surprising that we are able to see this behavior over a sample of 500 hedge funds with diverse styles (see Weinstein [2002]). Whatever is actually being done in reality, it is possible to replicate a pattern that looks like the pattern of overall hedge funds returns over the last ten years by hypothesizing that every hedge fund has simply written a put on the stock index combined with a residual return process producing a modest alpha. Typical investors would not want this investment profile. The short option pattern may also be found in non-systematic (alpha) exposures. One interesting example is hedge fund participation in syndicated catastrophe reinsurance (see Zuckerman and Francis [2004]). If the frequency and severity of hurricanes is in the normal range, the insurer/participant receives a steady profit; when the frequency or severity is much higher than normal, the insurer/participant experiences substantial losses associated with damage claims.

[2]From probability theory, we know that a constant value multiplied by a normal distribution is normal, and that a normal distribution plus a normal distribution is normal, as in the standard model, Equations (1) and (2). The product of a random distribution (of beta) and a normal distribution (market returns), as in Equations (3) and (4), generates returns that are not normal. In fact, 11 of the 13 hedge fund style indexes fail a commonly used test for normality applied to the monthly return

series. In general, the non-normal hedge fund indexes show negative skewness and excess kurtosis (fat tails), consistent with asymmetric beta exposures, and neither trait would be considered desirable by investors. In addition to non-normality, there is also autocorrelation in the return series in 7 of the 13 hedge fund styles. This too is undesirable, as the presence of autocorrelation makes multiperiod return series more volatile.

[3]This requirement is supported by a discussion of the general conditions required for setting investor preferences based on mean-variance portfolio analysis. In traditional portfolio analysis, it is assumed that asset returns are (approximately) multivariate-normal. Normality is not absolutely necessary, and more general conditions are sufficient that require only the class of (symmetric) multivariate elliptical distributions, where normal distributions are just a special case.

Elliptical distributions are useful in modeling return distributions with fatter tails than normal (leptokurtic), such as those observed with hedge fund returns. But for even the more general elliptical distributions, a requirement is symmetric dependence between asset returns described by a single covariance matrix (see Ingersoll [1987]). The symmetric dependence condition is violated if the hedge fund returns are more correlated in down markets than in up markets, as observed with the asymmetric betas for hedge fund styles in Exhibit 1.

Violating the symmetric dependence condition yields portfolios that are skewed even if the underlying asset returns are not skewed. Investors clearly care about skewness if they wish to distinguish between upside risk and downside risk, but skewed return patterns cannot be ranked by mean and variance measures alone. Therefore, traditional portfolio analysis is inadequate for hedge fund returns showing asymmetric dependence.

[4]Proprietary software is used to calculate the active efficient frontier for this example. It includes a manager structure module that calculates mean-variance efficient portfolios incorporating both misfit and residual risks.

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